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or

$$r = \tfrac{1}{2}\frac{m^2v^2}{(m + m')^2d}.$$

But $v^2 = 2gh$, and hence,

$$r = \frac{m^2gh}{(m + m')^2d}.$$

Finally,

$$F = (m + m')r = \frac{m^2gh}{(m + m')d}\text{ poundals} = \frac{m^2h}{(m + m')d}\text{ pounds.}$$

Substituting values,

$$F = \frac{500 \times 500 \times 10}{(500 + 400) \times \frac{1}{4}} = 11{,}111\tfrac{1}{9}\text{ lbs.}$$

Also solved by A. M. Harding and J. A. Caparo. Erroneously solved by Herbert N. Carleton.

305. Proposed by B. J. BROWN, Victor, Colorado.

A particle is to be projected so as to graze the top of a wall h feet high, at a distance of a feet from the point of projection, and to strike the ground at a distance b feet from the foot of the wall. Find the velocity of projection, and the inclination of the path to the horizontal, at the ground and at the top of the wall. I. C. S. 1903.

Solution by Emma M. Gibson, Assistant at Drury College.

The equation of the path of the particle is

$$y = x \tan \alpha - \tfrac{1}{2}g\frac{x^2}{\nu_0^2 \cos^2 \alpha},$$

where ν_0 is the initial velocity and α the angle of projection.

Knowing the points (a, h), $(a + b, 0)$ to be on the curve the following satisfied relations exist:

$$h = a \tan \alpha - \tfrac{1}{2}g\frac{a^2}{\nu_0^2 \cos^2 \alpha} \tag{1}$$

and

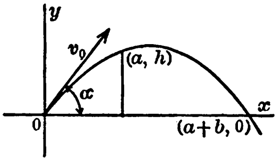
$$(a + b) \tan \alpha = \tfrac{1}{2}g\frac{(a + b)^2}{\nu_0^2 \cos^2 \alpha} = 0. \tag{2}$$

From equation (2),

$$\nu_0^2 = \frac{(a + b)g}{2 \sin \alpha \cos \alpha}, \quad \text{or} \quad \nu_0 = \sqrt{\frac{(a + b)g}{\sin 2\alpha}}.$$

Substituting this value of ν_0 in (1) and solving for $\tan \alpha$,

$$\tan \alpha = \frac{(a + b)h}{ab}.$$



Hence,

$$\alpha = \tan^{-1}\left[\frac{(a + b)h}{ab}\right],$$

the inclination of the path at the ground.

The equation of the tangent to the curve at any point (x_1, y_1) is

$$\frac{y + y_1}{2} = \frac{x + x_1}{2}\tan \alpha - \tfrac{1}{2}g\frac{x_1x}{\nu_0^2 \cos^2 \alpha}.$$

At (a, h) the tangent becomes

$$y + h = (x + a) \tan \alpha - \frac{agx}{\nu_0^2 \cos \alpha} = \left[\frac{\nu_0^2 \sin \alpha \cos \alpha - ag}{\nu_0^2 \cos^2 \alpha}\right]x + a \tan \alpha.$$

The inclination of this line,

$$\tan^{-1}\left[\frac{\nu_0^2 \sin \alpha \cos \alpha - ag}{\nu^2 \cos^2 \alpha}\right] = \tan^{-1}\left[\left(\frac{b - a}{ab}\right)h\right],$$

is the inclination of the path to the horizontal at the top of the wall.

Also solved by H. C. FEEMSTER, J. A. CAPARO, CLIFFORD N. MILLS, and HORACE OLSON. Erroneously solved by HERBERT N. CARLETON.

306. Proposed by EMMA M. GIBSON, Drury College.

A sphere is composed of a solid homogeneous hemisphere and a very thin hemispherical shell of equal mass. What is the greatest inclination of a rough plane on which the sphere can just rest in equilibrium?

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

Let the radius of the sphere be a, the center at O, the center of gravity at G; then

$$OG = \frac{1}{2}\left(\frac{a}{2} - \frac{3a}{8}\right) = \frac{a}{16};$$

however the sphere is placed, G is on a circumference of radius $a/16$, centered at O. If the sphere is in equilibrium, G is vertically over the contact, C, of the sphere with the inclined plane on which it rests; the inclination of the plane is the angle COG, which has its greatest value when CG is tangent to the little circle; this value is $\sin^{-1} 1/16$. Hence, the greatest inclination of the plane is $\sin^{-1} 1/16$ or $\tan^{-1} \mu$ (μ being the coefficient of friction) whichever happens to be the less.

Also solved by J. A. CAPARO. Erroneously solved by HERBERT N. CARLETON.

NUMBER THEORY.

226. Proposed by ELBERT H. CLARKE, Purdue University.

If $0!$ is taken equal to 1, and if k is any positive integer greater than or equal to 2, show that

$$\sum_{n=0}^{\infty} \frac{n!}{(k+n)!} = \frac{1}{k-1} \cdot \frac{1}{(k-1)!}.$$

I. SOLUTION BY S. A. JOFFE, New York City.

Since

$$\frac{n!}{(k+n)!} = \frac{1 \cdot 2 \cdot 3 \cdots n}{1 \cdot 2 \cdot 3 \cdots n(n+1)(n+2) \cdots (n+k)} = \frac{1}{(n+1)(n+2)\cdots(n+k)},$$

we see that

$$\sum_{n=0}^{\infty} \frac{n!}{(k+n)!} = \sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)\cdots(n+k)}. \tag{1}$$

Now, the series in the second member, each term of which is the reciprocal of a factorial expression, may be easily summed by an elementary theorem in finite differences or by the following equivalent simple method.

Noticing that

$$\frac{1}{(n+1)(n+2)\cdots(n+k-1)} - \frac{1}{(n+2)(n+3)\cdots(n+k)} = \frac{(n+k)-(n+1)}{(n+1)(n+2)\cdots(n+k)} = \frac{k-1}{(n+1)(n+2)\cdots(n+k)},$$

we find that

$$\frac{1}{(n+1)(n+2)\cdots(n+k)} = \frac{1}{k-1}\left[\frac{1}{(n+1)(n+2)\cdots(n+k-1)} - \frac{1}{(n+2)(n+3)\cdots(n+k)}\right].$$